22. Appendix J. Reference Sediment Motion

The reference condition that is most commonly used is where the non-dimensional transport rate, W^*, is equal to 0.002 (Parker, 1990).

$$W^* = \frac{(s-1)gq_s}{\rho_s(\tau_g/\rho)^{1.5}} = 0.002 \tag{1}$$

where s = relative specific density, g = acceleration of gravity, q_s = sediment transport rate, ρ_s = sediment density, τ_g = grain shear stress, ρ = water density. The transport rate, q_s, is primarily dependent upon the Shield's number, θ:

$$\theta = \frac{\tau_g}{\gamma(s-1)D_{50}} \tag{1}$$

where θ = dimensionless Shield's number; τ_g = grain shear stress; γ = specific weight of water; s = relative specific density of sediment; and D_{50} = mean sediment size. The Shields number that gives W^* = 0.002 is termed the reference Shield's stress (θ_r). It can be described as the condition when many particles are moving and there is a small, but measureable, sediment transport rate. In our analysis, it corresponds to a Shields number of 0.0386.

The total shear stress can be separated into grain shear stress and form drag. Grain shear stress is commonly understood to be responsible for bedload transport and the shear stress due to form drag is commonly ignored. The channel grain shear stress τ_g is calculated as

$$\tau_g = \gamma R' S \tag{2}$$

where R' = channel hydraulic radius due to grain shear stress; and S = friction slope. The total shear stress is partitioned into that due to form drag and that due to grain roughness. Manning's equation is valid for the channel hydraulic radius due to grain shear stress:

$$U = \frac{C_m}{n_g} R'^{\frac{2}{3}} S_f^{\frac{1}{2}} \tag{3}$$

where C_m = 1.0 for SI units, and 1.486 for English units, or $C_m = (g/9.81)^{\frac{1}{3}}$, and R' is the hydraulic radius due to grain shear stress . Dividing this equation by the Manning's equation gives:

$$\frac{R'}{R} = \left(\frac{n_g}{n}\right)^{1.5} \tag{4}$$

where R is the total hydraulic radius and n is the total Manning's roughness coefficient. The Manning's roughness coefficient for the bed grains, n_g, can be computed from the roughness height. First, the logarithmic velocity distribution is integrated over the depth to yield (López and Barragán, 2008):

$$\frac{U}{u_*'} = \frac{1}{\kappa}\ln\frac{R'}{k_s} + 6.25 \tag{5}$$

where κ is the von Karman constant (0.4), u_* is the shear velocity, and the log-law constant has assumed to be 6. Eq (5) can be approximately fit by the power law relation:

$$\frac{U}{u_*'} = 8.1\left(\frac{R'}{k_s}\right)^{\frac{1}{6}} \tag{6}$$

where k_s is a representative roughness height. Parker (1991) also used Eq (6) to approximate the roughness coefficient in gravel bed streams. The fit is best for R/k_s values between 5 and 200, which is the value most natural rivers will fall into. The error associated in predicting Manning's n values with this approximation is less than 3%.

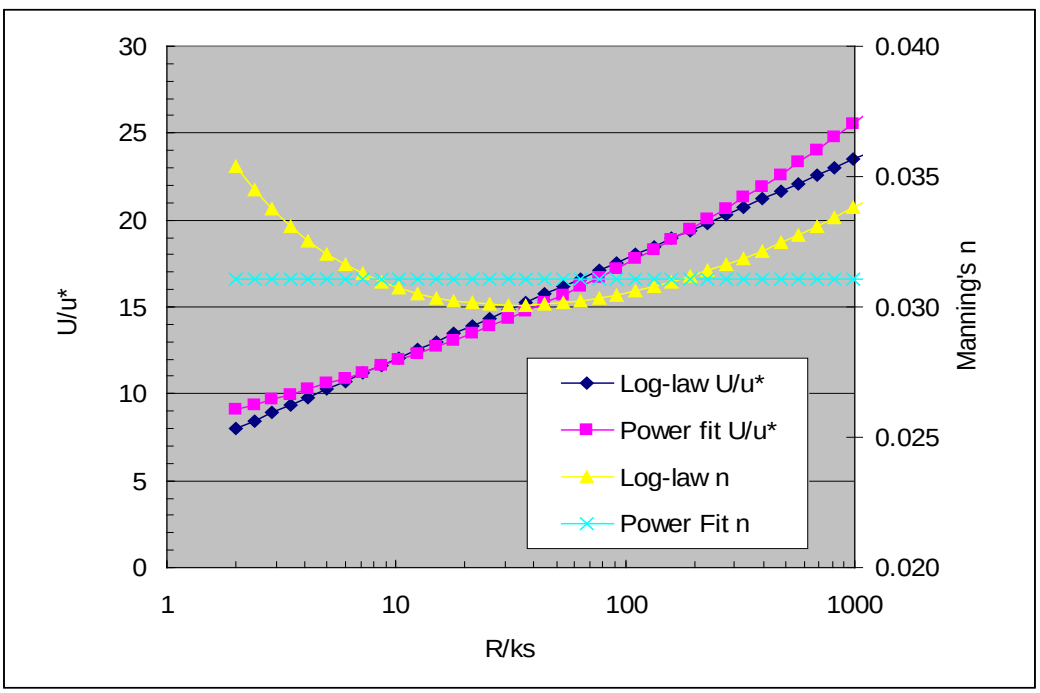

Figure 22-1. Comparison between Eq. 5 and 6. Also shown on the figure is the comparison between assuming k_s = 240 mm.

The Manning's roughness coefficient due to grain shear, n_g, can then be computed from the roughness height using the following dimensionally consistent formula:

$$n_g = 0.058\left(k_s / g\right)^{\frac{1}{6}} \tag{7}$$

Several different relations in alluvial rivers have been proposed for k_s ranging from 0.95 D_{50} (Federal Highway Administration, 1975) to 3 D_{90} (van Rijn, 1982). A more recent publication, López and Barragán (2008), suggests that 2.4D_{90}, 2.8D_{84}, and 6.1D_{50} all give equivalent predictions of Manning's Roughness coefficient for river beds with gravel size or larger sediment, with a nonsinuous alignment and a flow path free of vegetation or obstacles. In their publication, they use the log law approximation (5) to compute Manning's n, but as shown above, the error associated with using the power fit approximation (6) is less than 3 %.